Journal on
Perceptual Control Theory
Vol 1 No 1, 1999

A PCT Model of Human Oculomotor Control
Part 1: Basic Outlines

by
Wolfgang Zocher
Private Researcher

Last Revision Date: March 21, 1999 ISSN 1437-9325

Abstract

This paper is the start of a series of articles presenting a model of human oculomotor control.What matters most in this first part is the development of a basic model of the horizontal eye movement apparatus based on PCT (*perceptual control theory*) by William T. Powers [Powers, 1973] which allows to relate the mathematical description of the model very close to the physiological and neurological reality. The resulting model is used throughout the series as a basic component in the simulation of all kinds of eye movement.

Contents

1 Introduction

In the past two or three decades the horizontal eye movement system has been studied in detail by many investigators. Essentially all this research was based on phenomological models of eye movements with the same basic structure: a mechanical model of opposing muscles attached to the globe (see e.g. [Carpenter, 1977]).
In Figure 1 a model of this kind is shown.

Mechanical Model of Horizontal Oculomotor System

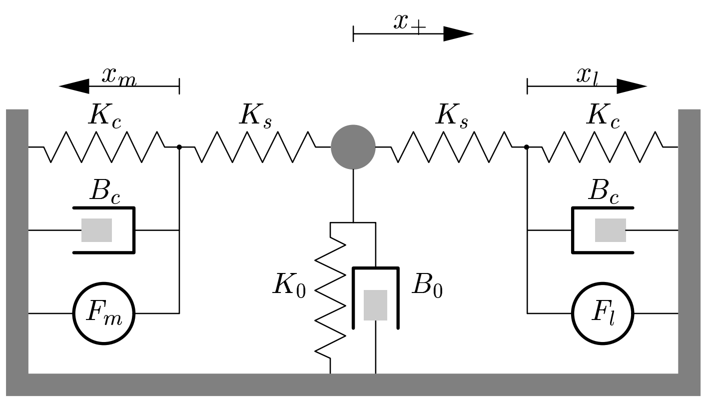


Figure 1

Here, each muscle of the horizontal pair of muscles is modelled as a serial spring representing the elasticity of the muscle, cascaded with a parallel combination of a force generator a (nonlinear) viscous element and a spring. The force generator and the dashpot element represents the contractive part of the muscle. The arrangement of springs and dashpots varies with the models.

The development was guided by the measurement of steady-state, macroscopic muscle properties, primarily in human strabismus patients ([Collins, 1975], [Robinson et al., 1969]). These models can generate realistic position, velocity and developed tension for different kinds of eye movements (fixation, pursuit and saccadic movement).

The dynamic properties of this kind of mechanical models can be expressed with a set of differential equations. For the *medial rectus* in the model of Figure 1 the equations are obtained by analyzing the forces applied by the components of the mechanical circuitry:
The tension developed by the three elements in parallel is:

$$T_m(t) = B_c\,(\frac{\partial}{\partial t}\,x_m(t)) + K_c\,x_m(t) + F_m(t) \qquad (1.1)$$

The serial spring is prolongated by $x + x_m$, so the force developed is:

$$T_m(t) = K_s\,(x(t) + x_m(t)) \tag{1.2}$$

Putting both results for the tension together by equating both expressions gives the differential equation for the medial rectus. The other parts of the model are descibed in a similar way; the resulting three differential equations are:
Medial Rectus:

$$B_c\,(\frac{\partial}{\partial t}\,x_m(t)) + K_c\,x_m(t) + F_m(t) = K_s\,(x(t) + x_m(t)) \tag{1.3}$$

Lateral Rectus:

$$B_c\,(\frac{\partial}{\partial t}\,x_l(t)) + K_c\,x_l(t) + F_l(t) = K_s\,(x_l(t) - x(t)) \tag{1.4}$$

Elasticity and viscosity of orbit:

$$B_o\,(\frac{\partial}{\partial t}\,x(t)) + K_o\,x(t) = (-2\,x(t) - x_l(t) + x_m(t))\,K_s \tag{1.5}$$

This is a coupled system of first order DEQ's, coupled by the deviation of the eyeball x. This system can be solved numerically or analytically.

Solving the DEQ's for the first derivatives of x_m, x_l and x shows that there is a control structure behind those equations. As an example the DEQ 1.1 for the medial rectus is transformed:

$$T_m(t) = B_c\,(\frac{\partial}{\partial t}\,x_m(t)) + K_c\,x_m(t) + F_m(t) \tag{1.1}$$

gives:

$$\frac{\partial}{\partial t}\,x_m(t) = \frac{1}{B_c}\,(T_m(t) - K_c\,x_m(t) - F_m(t)) \tag{1.1$'$}$$

This equation describes an integrator with negative feedback of

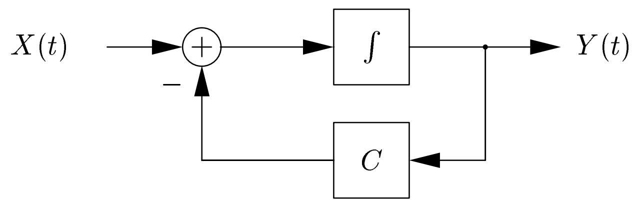
$$\frac{K_c}{B_c}\,x_m(t)$$

and an input of

$$\frac{F_m(t) - T_m(t)}{B_c}$$

or, in terms of control theory, a *proportional element with delay* or *leaky integrator*. See Figure 1a.

The Integrating Device



$$\frac{\partial}{\partial t} Y(t) = X(t) - C\,Y(t)$$

Figure 1a

The main question which arises now is:

Where comes the entity T_m from?

Assumed, that the entity of developed tension of the muscle is available, the model described above works. Because of the nature of the model as a phenomological one the origin of T_m is'nt in the scope of interest.

Going over to a control theoretical model, intimated by the transformation of the DEQ which describes the medial rectus, it is assumed that **all** physical entities involved in the control process are measurable or computable by other measurable entities. This assumption is one of the basics of perceptual control theory: the concept of the *controlled variable*.

The major task described in this paper is the finding of controlled variables in human oculomotor control and to refine the mechanical model in Figure 1.

2 Basic Outlines of the PCT Model

In his Minireview on *Proprioceptive Knowledge of Eye Position* Martin J. Steinbach [Steinbach, 1987] gives a condensed overview of eye muscle proprioception. Without any doubt there are muscle spindles [Lukas et al., 1994] to perceive changes of lenghts of muscles and Golgi tendon organs [Eggers, 1982] to perceive developed tension in the eye muscles of man.

By analogy with the limb-muscle receptors, the signals of these receptors may serve as feedback signals used in the control of eye movement. However, this possibility has commonly been discounted. Despite this fact and in consideration of latest results of the investigations of Hayman, Dutia, Knox and Donaldson ([Hayman et al., 1993], [Knox and Donaldson, 1993]) the signals of both types of receptors will be used in modelling the human horizontal eye movement system.

Moving the eye requires the combined action of at least two muscles: an *agonist* which contracts and develops the force required to rotate the eye in his socket and an *antagonist* which relaxes and gets longer. This is the picture of reciprocal innervation of an opposing muscle pair.

In the following the opposing pair of muscles which drive the right eye horizontally is regarded. These muscles are the *right medial rectus (RMR)* pulling the eyeball towards the nose when active and the *right lateral rectus (RLR)* which pulls the eyeball in opposite direction towards the right temple.

To simplify modelling, this pair of muscles is arranged in a straight line with the eyeball as a movable point just between them. This translatoric movement is equivalent to rotation under the assumption that the eyeball is a sphere.

Each muscle is treated as a two-component system, an elastic element and a contractile element. The contractile element is the active element: innervation leads to shorten the whole muscle.

The elasticity and all velocity dependent viscosities of the eyeball are modeled by a passive damping element; this is in no way unusual in correspondance to the model shown in Figure 1 and only describes the physical properties of an eye muscle by means of a mechanical analogon. For a detailed description of muscle models see [McMahon, 1984].

Taking into account the receptors for tension and length changes in the muscle, the Golgi tendon organ and the muscle spindle, the first step is done to get a PCT-model of oculomotor control. See Figure 1b

Proprioreceptive elements in eye muscle

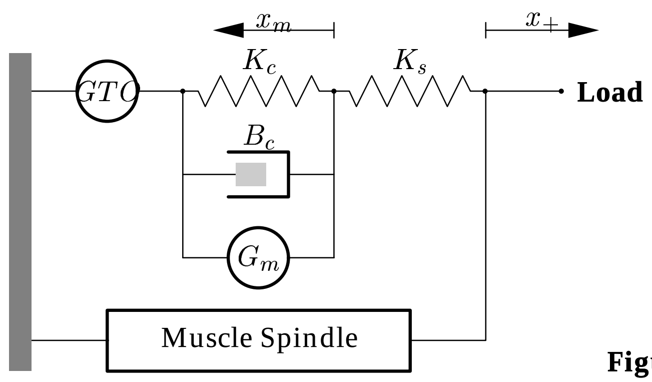


Figure 1b

Both receptors report their result of measurement in form of neural signal, spike trains with a frequency of spikes proportional to the magnitude of the measured entity. Those signals are called *proprioceptive signals*. See [Powers, 1973], chapter 3 for a detailed description of neural signals. Here and throughout the paper, proprioceptive signals are denoted by p with appropriate subscripts to show the origin.

Innervating the muscle by a nervous signal $\alpha_m(t)$ causes the the muscle to contract, stretching the attached tendon and stimulating the Golgi cells, sensory receptors clustered on and near the tendon. These receptive cells deliver a neural signal proportional to the force developed by the muscle.

In real muscles the process of innervation is initiated by the motor neuron which receives two input signals:

- a command signal $f_m(t)$ (excitatory), and
- a proprioceptive tendon feedback signal $p_{Tm}(t)$ (inhibitory).

$$\alpha_m(t) = f_m(t) - p_{Tm}(t) \tag{2.1}$$

This equation describes the computation of an innervation signal out of two proprioceptive signals. Since these signals represent forces,

$$A_m(t) = F_m(t) - T_m(t) \tag{2.2}$$

describes the comparative function for the forces. Figure 1c shows the signal paths.

Tension control in the medial rectus
(negative tension feedback loop)

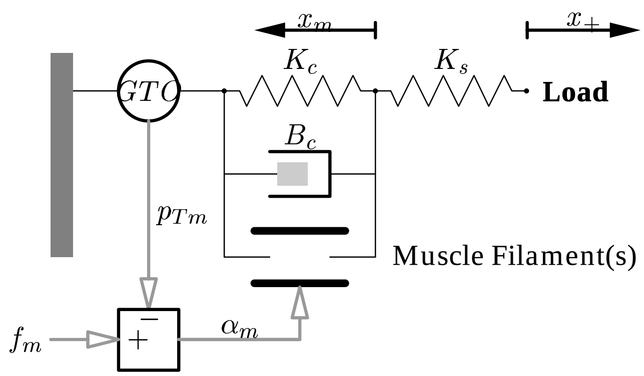


Figure 1c

The shortening of the muscle fibers $x_m(t)$ as the primary process happening in the contractile element generates a force proportional to $\alpha_m(t)$; the viscosities are modeled by a dashpot element. Dashpot elements develop zero force when they are stationary, but resist lenght changes with a force $F = B\,\frac{\partial}{\partial t}\,x(t)$. B may be either a constant or a function of $\frac{\partial}{\partial t}\,x(t)$. Here and throughout the different viscosities in the muscles and the damping of the eyeball are assumed to be constant.
The parallel elastic component K_c is used to model the connective tissue surrounding the muscle fibers.
So the force generated by the contractile element and the parallel elastic element becomes:

$$A_m(t) = K_c\,x_m(t) + B_c\,\frac{\partial}{\partial t}\,x_m(t) \tag{2.3}$$

and

$$\frac{\partial}{\partial t}\,x_m(t) = (A_m(t) - K_c\,x_m(t))/B_c \tag{2.4}$$

The force acting on the load point is determind by the position of the load point and by the amount of contraction of the contractile element. The lenght difference is (in the coordinate system used in Figure 1c) is

$$\Delta x_{sm} = x_m(t) + x(t) \tag{2.5}$$

The force applied to the load point, in other words, the tension developed by the muscle is:

$$T_m(t) = K_s \Delta x_{sm} \tag{2.6}$$

The right hand side of equation 2.5 is the equivalent to the measured tension in a real muscle. This expression is only needed to perform a mathematical model - in nature this value for $T_m(t)$ is measured by the Golgi tendon organ. So we don't need the lenght of the muscle $x(t)$ measured to make the tension control loop work.
The equations 2.2, 2.4, 2.5 and 2.6 describe the closed loop for the tension control in the medial rectus. The only input to the loop is the required tension $F_m(t)$, the *reference value* for the tension. The corresponding equations for the lateral rectus are:

$$A_l(t) = F_l(t) - T_l(t) \tag{2.2'}$$

$$\frac{\partial}{\partial t}\, x_l(t) = \frac{A_l(t) - K_c\, x_l(t))}{B_c} \tag{2.4'}$$

$$\Delta x_{sl} = x_l(t) - x(t) \tag{2.5'}$$

$$T_l(t) = K_s \Delta x_{sl} \tag{2.6'}$$

The tension developed by both of the muscles act on the eyeball as opposing forces. The resulting force:

$$T_o = T_l - T_m \tag{2.7}$$

has to move the eyeball against the resistance of certain elasticities and viscosities; the final equation describing the movement of the eyeball dependent from resulting force T_o is:

$$\frac{\partial}{\partial t}\, x(t)) = \frac{T_o - K_o\, x(t)}{B_o} \tag{2.8}$$

Now, the description of the horizontal eye movement system is complete. It's a good practice in control theory to represent complex mathematical descriptions as diagrams of interconnected functional blocks. Inspection of the equations for both of the muscles and the eyeball shows that we need three types of blocks:

- summing points (as in equation 2.2, 2.5, 2.7)
- G_2: proportional function blocks P-blocks (as in equation 2.6)
- G_1,G_3: leaky integrator blocks PT_1-blocks (as in equation 2.4, 2.8)

The resulting diagram is shown in Figure 2.

Block diagram of horizontal eye movement model

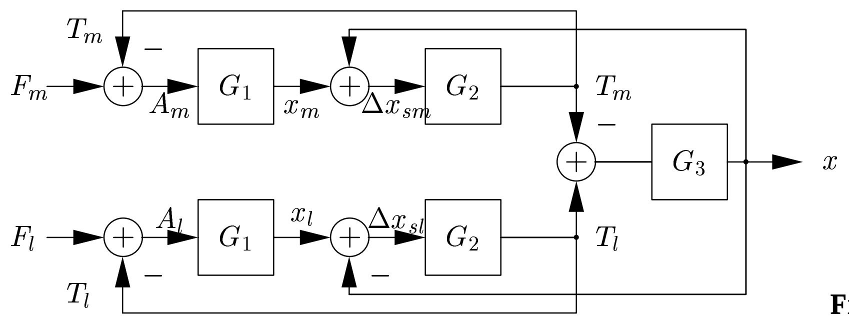


Figure 2

3 Simulating the system

There are **many** ways to simulate the system developed throughout the previous section on a computer. The easiest way is to use a program which offers the ability to solve mathematically the complete set of equations with respect to dedicated variables. Another type of programs allows to construct a block diagram like Figure 2 which is simulated when finished. Programming the simulation in some programming language or other is the hardest way and should be avoided. At the time, the author uses the symbolic algebra system *Maple* [Redfern, 1996] to get analytical closed form solutions. Future work will be done with *Scicos* [R. Nikoukhah, 1998].

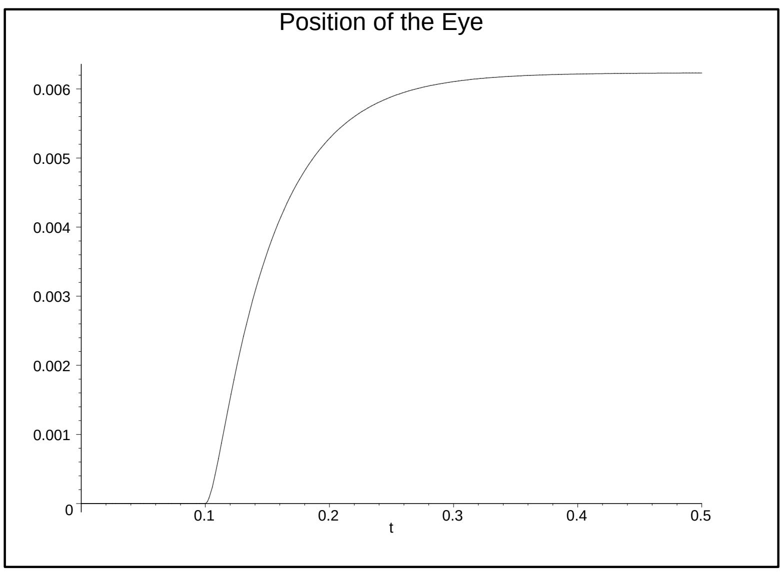
To simulate the system all parameter values are needed. For a model like the one described in Figure 1, [Collins et al., 1975] has determind the spring constants for the parallel and the serial spring by implanting a little force detector into the eye muscle. The resulting values are used rescaled. Since Collins in all his models used a nonlinear (i.e. velocity dependent) viscosity in the contractile element, this parameter was estimated by regression, comparing simulation results with published data on developed tension with respect to different positions of the eyeball. The assumption of a dashpot element with linear chracteristic holds for the operational region of the muscle [Enderle et al., 1991].
The parameters used in the simulations are:

$$
\begin{array}{lcrl}
K_c & = & 61.32 & [N/m] \\
K_s & = & 127.70 & [N/m] \\
K_o & = & 25.54 & [N/m] \\
\hline
B_c & = & 2.00 & [N\,s/m] \\
K_o & = & 3.07 & [N\,s/m]
\end{array}
$$

In control theory the behavior of control systems is tested with standardized reference functions, for example with a step to "1" performed at time t_1. So, as a first test of the system, $F_m(t)$ is set constant zero, $F_l(t)$ is the step function just mentioned with t_1 set to $0.1\,[s]$. A jump to 1.0 for $F_l(t)$ at time $t = t_1$ means, that the lateral rectus should suddenly develop the required force of $1.0\,[N]$. The initial values for the simulation are:

$$
\begin{array}{lcl}
x(0) & = & 0.00\,[m] \\
x_m(0) & = & 0.00\,[m] \\
x_l(0) & = & 0.00\,[m]
\end{array}
$$

The resulting movement of the eyeball in $[mm]$ is shown in Figure 3.



4 Conclusions

The simple simulation of eye movement at the end of the previous section only shows that the system works in any way. The next article in this series will go more into the details of this result determining the length-tension characteristics of the muscles in fixation experiments: the simple motor control system will be extended by a position control system, which also allows to perform saccades.

References

[Carpenter, 1977] Carpenter, R. (1977). *Movement of the Eyes*. Pion, London.

[Collins, 1975] Collins, C. C. (1975). The human oculomotor control system. In Lennerstrand, G. and y Rita, P. B., editors, *Basic Mechanisms of Ocular Motility and their Clinical Implications*, pages 145–180. Pergamon Press, Oxford.

[Collins et al., 1975] Collins, C. C., O'Meara, D., and Scott, A. B. (1975). Muscle tension during unrestrained human eye movement. *J. Physiol.*, 245:351–369.

[Eggers, 1982] Eggers, H. M. (1982). Functional anatomy of the extraocular muscles. In Duane, T. D., editor, *Biomedical Foundations of Ophthalmology*, pages 1–52. Harper and Row, Philadelphia.

[Enderle et al., 1991] Enderle, J. D., Engelken, E. J., and Stiles, R. N. (1991). A comparison of static and dynamic characteristics between rectus eye muscles and linear muscle model predictions. *IEEE Transcations on Biomedical Engeneering*, 38(12):1235–1245.

[Hayman et al., 1993] Hayman, M., Dutia, M., and Donaldson, I. (1993). Afferent signals from pigeon extraocular muscle modify the activity of neck muscles during the vestibulocollic reflex. *Proc. R. Soc. Lond.*, 253:122–155.

[Knox and Donaldson, 1993] Knox, P. and Donaldson, I. (1993). Do extraocular muscle afferent signals play a role in the human vestibulo-ocular reflex? *Soc. Neuroscience Absts.*, 19:858.

[Lukas et al., 1994] Lukas, J. R., Aigner, M., Blumer, R., Heinzl, H., and Mayr, R. (1994). Number and distribution of neuromuscular spindles in human extraocular muscles. *Invest. Opth. and Visual Science*, 35:4317–4327.

[McMahon, 1984] McMahon, T. A. (1984). *Muscles, Reflexes, and Locomotion*. Princeton University Press, Princeton, NJ.

[Powers, 1973] Powers, W. T. (1973). *Behavior: The Control of Perception*. Aldine de Gruyter, Hawthorne, NY.

[R. Nikoukhah, 1998] R. Nikoukhah, S. S. (1998). *SCICOS - A Dynamic System Builder and Simulator: Users Guide*. INRIA.

[Redfern, 1996] Redfern, D. (1996). *The Maple Handbook.* Springer Verlag, New York.

[Robinson et al., 1969] Robinson, D. A., O'Meara, D. M., Scott, A. B., and Collins, C. C. (1969). Mechanical components of human eye movements. *J. Appl. Physiol.*, 26:548–553.

[Steinbach, 1987] Steinbach, M. J. (1987). Proprioceptive knowledge of eye position. *Vision Research*, 26(10):1737–1744.